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ABSTRACT

In the standard Big Bang cosmology the canonical value for the ratio of relic neutrinos to CMB photons is 9/11. Within the framework of the Standard Model of particle physics there are small corrections, in sum about 1%, due to slight heating of neutrinos by electron/positron annihilations and finite-temperature QED effects. We show that this leads to changes in the predicted cosmic microwave background (CMB) anisotropies that might be detected by future satellite experiments. NASA's MAP and ESA's PLANCK should be able to test the canonical prediction to a precision of 1% or better and could confirm these corrections.

Introduction. Neutrinos are almost as abundant as photons in the Universe and contribute almost as much energy density [1]. Under the assumption that neutrinos decoupled completely before electrons and positrons annihilated (at a time of around 1 sec), the ratio of the number density of neutrinos to that of photons is

$$\frac{n_\nu}{n_\gamma} = \left(\frac{3N_\nu}{11}\right), \qquad (1)$$

where $N_\nu = 3$ is the number of neutrino species. Further, because of the heating of the photons by e^+/e^- annihilations, the ratio of the neutrino temperature to the photon temperature is $(4/11)^{1/3} = 0.714$. It follows that the ratio of the energy density of neutrinos to that of photons is

$$\frac{\rho_\nu}{\rho_\gamma} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_\nu = 0.681. \qquad (2)$$

It has been pointed out that the assumption that neutrinos decoupled completely before e^+/e^- annihilations is not precisely valid [2]. There is now a consensus that the neutrinos share in the heating somewhat, so their number and energy density is slightly larger than the canonical values, Eqs. (1, 2). The increase is equivalent to having slightly more than three neutrino species and the canonical ratios. (This is just a heuristic device, of course; the actual number of generations is fixed at three.) The change in the effective number of neutrino generations is [2, 3, 4, 5, 6, 7, 8]

$$\delta N_\nu^{\mathrm{ID}} = 0.03. \qquad (3)$$

The first calculations [2, 3, 4] of this effect were "one-zone" estimates that evolved integrated quantities through the process of neutrino decoupling. More refined "multi-zone" calculations tracked many energy bins, assumed Boltzmann statistics and made other approximations [5, 6]. The latest refinements have included these small effects as well [7, 8]. (A very recent calculation makes no approximations whatever and tracks the neutrino momentum distribution over 8 orders of magnitude in momentum [9], and arrives at slightly higher value, $\delta N_\nu^{\mathrm{ID}} = 0.045$. Until the discrepancy is understood we will stick with the earlier estimates; in any case it is simple to rescale our results.)

There is another effect operating at roughly the same time which acts in the same direction; it involves finite-temperature QED corrections to the

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energy density of the electron, positron and photon portion of the plasma due to interactions [10, 11]. This effect decreases the energy density of the $e^{\pm}\gamma$ plasma. Consequently this reduces the amount of energy converted to photons when electrons and positrons annihilate thereby slightly raising the ratio of the neutrino to photon energy densities. This QED effect can also be expressed as an increase in the number of neutrino species [10, 11]

$$\delta N_{\nu}^{\mathrm{QED}} = 0.01. \tag{4}$$

Together, incomplete annihilation and QED finite-temperature corrections lead to an increase in the neutrino energy density over the canonical value by slightly more than 1%, corresponding to

$$\delta N_{\nu} = 0.04. \tag{5}$$

These two corrections were initially considered in the context of Big Bang Nucleosynthesis. Their net effect is to increase the predicted ^{4}He abundance by a tiny amount, $\Delta Y_P = +5 \times 10^{-4}$, which given the present observational uncertainties, $\sigma_{Y_P} \geq 10^{-2}$, is undetectable and likely to remain so for quite some time. (The quantity Y_P denotes the primordial mass fraction of ^{4}He.)

On the other hand, the small increase in the neutrino energy density can have a significant – and potentially detectable effect – on another remnant of the Big Bang – the cosmic microwave background (CMB). In particular, the anisotropies in the CMB are very sensitive to the epoch of matter-radiation equality, which depends on the neutrino energy density. As we shall show, the sensitivity is so great that by itself the additional energy density in neutrinos should be detectable by the very precise measurements of the CMB anisotropy that will by made the two forthcoming satellite experiments, NASA's MAP and ESA's PLANCK Surveyor. However, the situation is complicated somewhat by the fact that predictions for the CMB anisotropies also depend on other cosmological parameters.

The aim of this paper is to address quantitatively the detectability of the small increase in neutrino energy density due to incomplete decoupling and finite-temperature QED effects. For definiteness, we assume that the primordial density perturbations that seed structure formation and lead to CMB anisotropy were set during inflation and allow five other parameters to vary. They are: baryon density (Ω_B); Hubble constant (H_0); amplitude of primordial perturbations; slope of primordial perturbations (n); and epoch

of reionization. We find that (i) if these other parameters are held fixed (e.g., because they are determined by other measurements), then detectability is a sure thing; and (ii) if the other parameters are allowed to vary, then the situation is less promising; but, assuming non-linear effects are not a serious contaminant and polarization of the CMB anisotropy is also measured with precision[12], these small corrections should be detectable.

Probing Neutrino Physics with the CMB. Anisotropies in the CMB are best characterized by expanding the temperature field on the sky in terms of spherical harmonics:

$$T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi). \tag{6}$$

A given theory, specified by the primordial spectrum of perturbations and cosmological parameters, makes predictions about the multipole amplitudes, the a_{lm}'s. The predictions take the form of statements about the distribution of the a_{lm}'s. Inflationary theories typically predict that each of these coefficients is drawn from a Gaussian distribution; as such, the distribution can be defined by its variance. Thus, the fundamental predictions of inflationary models are

$$C_l \equiv \langle a_{lm} a_{lm}^* \rangle. \tag{7}$$

Much effort has gone into computing the C_l's over the last few years; they can be calculated very accurately once the cosmological parameters are chosen [13]. Viewed simplistically, the results of a CMB experiment are estimates of the C_l's, with errors given by ΔC_l. Then, by minimizing a χ^2 statistic

$$\chi^2 (\{\lambda_i\}) \equiv \sum_{l=2}^{\infty} \frac{\left(C_l (\{\lambda_i\}) - C_l^{\text{estimate}} \right)^2}{(\Delta C_l)^2}, \tag{8}$$

the underlying set of unknown cosmological parameters $\{\lambda_i\}$ can be estimated.

Of course, we cannot know in advance the values of C_l's that a given experiment will measure; however, by knowing what we expect for the ΔC_l's, we can estimate how large the uncertainties in the parameters should be ("error forecasting"). To do this, we assume that the measured C_l's will be close to the true C_l's. Then, by expanding χ^2 around its minimum at $\{\lambda_i^{\text{true}}\}$ we can estimate the precision to which a parameter can be determined (for

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further discussion of "error forecasting" in parameter estimation, see e.g., Refs. [14]):

$$
\begin{aligned}
\chi^2\left(\{\lambda_i\}\right) \quad &\simeq \chi^2\left(\left\{\lambda_i^{\text{true}}\right\}\right) + \frac{1}{2}\left.\frac{\partial^2\chi^2}{\partial\lambda_i\partial\lambda_j}\right|_{\lambda=\lambda^{\text{true}}}\left(\lambda_i - \lambda_i^{\text{true}}\right)\left(\lambda_j - \lambda_j^{\text{true}}\right) \\
&\equiv \chi^2\left(\left\{\lambda_i^{\text{true}}\right\}\right) + C_{ij}\left(\lambda_i - \lambda_i^{\text{true}}\right)\left(\lambda_j - \lambda_j^{\text{true}}\right).
\end{aligned}
\tag{9}
$$

The second-derivative (Fisher) matrix C_{ij} carries information about how quickly χ^2 increases as the parameters move away from their true values. Therefore, under some reasonable assumptions [15], the uncertainties in the parameters are determined by this matrix. We are interested only in the parameter[1] N_ν. If all the other cosmological parameters are held fixed, it is a simple exercise to show that its variance is given by

$$
\sigma_{N_\nu}^2 = \frac{1}{C_{N_\nu,N_\nu}}
\tag{10}
$$

If all other parameters are allowed to vary, then

$$
\sigma_{N_\nu}^2 = (C^{-1})_{N_\nu,N_\nu}
\tag{11}
$$

To proceed we need to specify

- **Cosmological Model.** For definiteness, we take this to be a Cold Dark Matter model with Hubble constant $H_0 = 50\,\text{km}\,\text{sec}^{-1}\,\text{Mpc}^{-1}$, baryon density $\Omega_B = 0.08$, COBE-normalized spectrum of scale invariant density perturbations (i.e., power-law index $n = 1$), no reionization, and energy density in cold dark matter particles $\Omega_{\text{CDM}} = 1 - \Omega_B = 0.92$. (We assume the simplest inflationary prediction of $\Omega = 1$.)

- **Experimental Errors.** Instead of tying ourselves to a particular experiment, we assume that the experimental uncertainty is given by

$$
\Delta C_l = \begin{cases} \sqrt{\frac{2}{2l+1}}C_l & l \leq l_{\max} \\ \infty & l > l_{\max} \end{cases}.
\tag{12}
$$

---

[1] The only other parameter estimation paper we are aware of which considers N_ν as a free parameter is Jungman et al. [14]. Their analysis, performed several years ago, only considered $l \lesssim 1000$ (then considered optimistic). Further they did not consider polarization. Where it is possible to compare with them, our results agree.

The error $[2/(2l+1)]^{1/2}C_l$ is the smallest possible given that each multipole amplitude a_{lm} can be sampled only $2l+1$ times; it is the irreducible sampling or *cosmic* variance. Equation (12) is obviously a simplification, but we have found it to be a reasonable approximation to the more realistic formula [16] which also accounts for detector noise. Further, it allows us to display our results as a function of l_{max}, which will give a clear sense of what angular scales need to be probed. We use a similar formula for polarization (with different l_{max}). For orientation, MAP is characterized by $l_{max} \simeq 1000$ and PLANCK by $l_{max} \simeq 2500$.

- **Model Parameters.** We allow for variation in five parameters besides the neutrino energy density: overall amplitude of the spectrum of density perturbations, epoch of reionization parametrized by the optical depth back to last scattering τ, H_0, Ω_B, and n.

Figure 1 shows the angular power spectrum (C_l vs. l) and how it changes as each of the six parameters is varied. Figure 2 shows the same for the polarization power spectrum. Using these derivatives, we can evaluate the Fisher matrix and compute the expected error in N_ν. Our results are summarized in Fig. 3.

The lower set of curves in figure 3 show the one-sigma errors on N_ν if all the other parameters are known. Even without polarizarion information, both PLANCK and MAP will detect the predicted $\delta N_\nu \sim 0.04$. If the other parameters are not well determined by other considerations, even a very high resolution temperature anisotropy experiment will have difficulty detecting the small predicted increase in N_ν. However, as Fig. 3 illustrates, with polarization information, the prospects are considerably brighter.

We should note here that other effects may also mimic a change in the effective number of neutrino species- particularly any field or particle with a relativistic equation of state at the matter-radiation equality epoch. For example, the presence of a random magnetic field will contribute to the relativistic energy density of the universe. For a given average field strength B_{eq} at the radiation-matter equality epoch, one could misconstrue this as an effective change in the number of neutrino species:

$$\delta N_\nu^{\mathrm{Mag.}} \approx 0.03 \left(\frac{B_{eq}}{10 \, \mathrm{gauss}} \right)^2 (\Omega_0 h_{50}^2)^{-4}, \qquad (13)$$

where h_{50} is the Hubble constant normalized to 50 km sec^{-1} Mpc^{-1}. If for example we constrain $\delta N_\nu^{\text{Mag.}} < .01$ this translates to $B_{\text{eq}} < 6$ gauss $(\Omega_0 h_{50}^2)^2$. Assuming that the magnetic field $B \propto a^{-2}$, where a is the scale factor, then this limit is an order of magnitude greater than the Big Bang Nucleosynthesis limit [17]. More importantly, a magnetic field this of this order may be measured or ruled out by Faraday rotation of the CMB [18] and perhaps other CMB measurements [19, 20], thus the magnetic field effect may be disentangled from δN_ν.

Concluding Remarks. As our analysis shows, future, high-precision CMB anisotropy measurements have the potential to measure the cosmic energy density in neutrinos to a precision of 1% or better. Such a measurement would have significant implications:

- If $N_\nu = 3$, further evidence for the existence of the tau neutrino. Note, the tau neutrino has yet to be directly detected in the laboratory.

- Determination that "$N_\nu = 3$" by CMB anisotropy would confirm the canonical assumption for the energy density in relativistic particles at the epoch of big-bang nucleosynthesis, which is an important input parameter for these calculations.

- Confirmation of the standard cosmology prediction that $T_\nu/T_\gamma = (4/11)^{1/3}$ to better than 1%. This would test the physics of e^+/e^- annihilation and neutrino decoupling in the early Universe.

- Confirmation of two small physics effects that together increase the cosmic neutrino energy density by about 1%. In particular, this would be the first evidence for finite-temperature QED corrections and a constraint to the strength of neutrino interactions in the early Universe.

- If a deviation from the expected $N_\nu = 3.04$ is found, evidence for additional relativistic particle species (or magnetic field) present in the early Universe or new physics in the neutrino sector (e.g., neutrino mass or decay) [21]. This would have significant implications for big-bang nucleosynthesis, structure formation in the Universe, and elementary-particle physics.

Realizing the full potential of the CMB as a probe of the cosmic neutrino backgrounds will require precision polarization and anisotropy maps

out to multipole number 3000. This seems very ambitious and perhaps even unattainable. Nonetheless, the potential payoff discussed here makes the goal worth striving for. If we have learned nothing else in the years since COBE, we have certainly learned that the experimenters have consistently manage to surprise theorists by achieving more than was once thought reasonable.

Finally, we should acknowledge that there is room to improve upon our analysis. For example, we have assumed only six cosmological parameters and ignored prior information about them. We have not considered the adverse role that "secondary anisotropies" that are generated at late times might play. There is clearly room for more work on this important subject.

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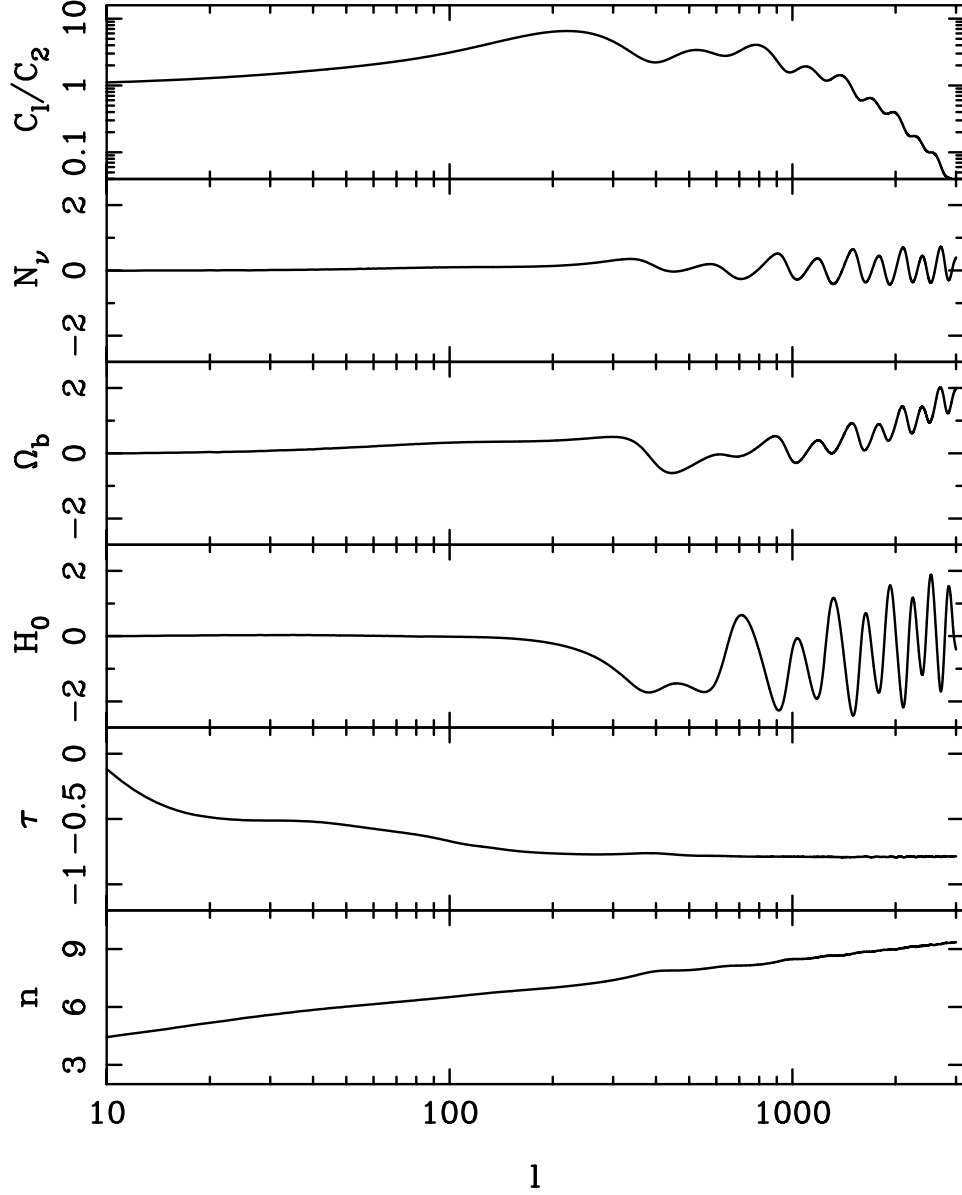[21] S. Dodelson, R. Lopez, R. Scherrer, and M. S. Turner, *in preparation.*

Figure 1: Temperature anisotropies and their derivatives. The top pane shows the quadropole-normalized CMB anisotropy spectrum as a function of multipole moment l, for our baseline CDM model: $N_\nu = 3$, $\Omega_B = 0.08$, $\Omega_{CDM} = 0.92$, $H_0 = 50 \mathrm{km\,sec^{-1}\,Mpc^{-1}}$, scale-invariant primordial perturbations, and no reionization. The lower panes show the derivatives of the C_l's with respect to the model parameters θ_i, $\partial \ln C_l / \partial \ln \theta_i$. For the optical depth to reionization parameter, $\partial \ln C_l / \partial \tau$ is plotted.
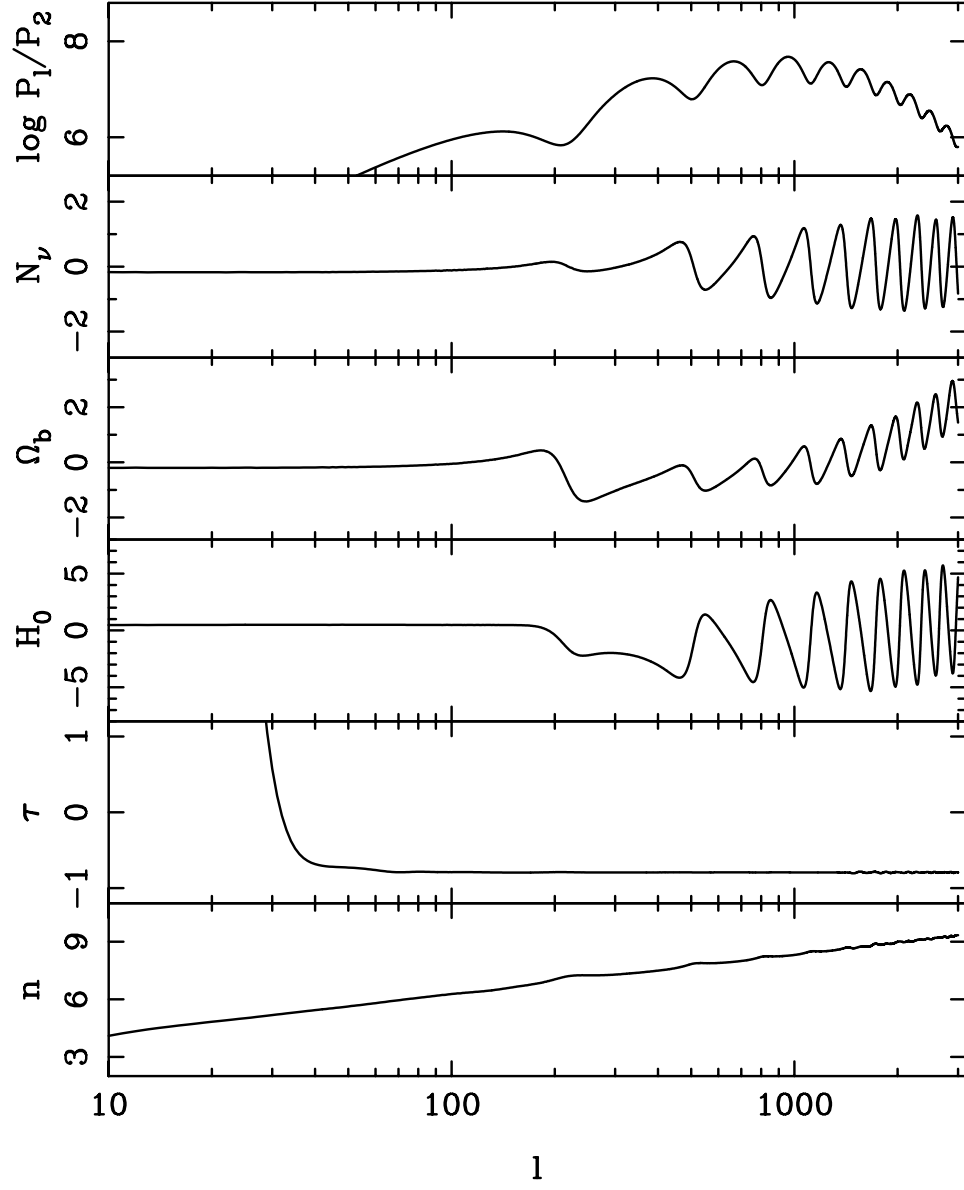
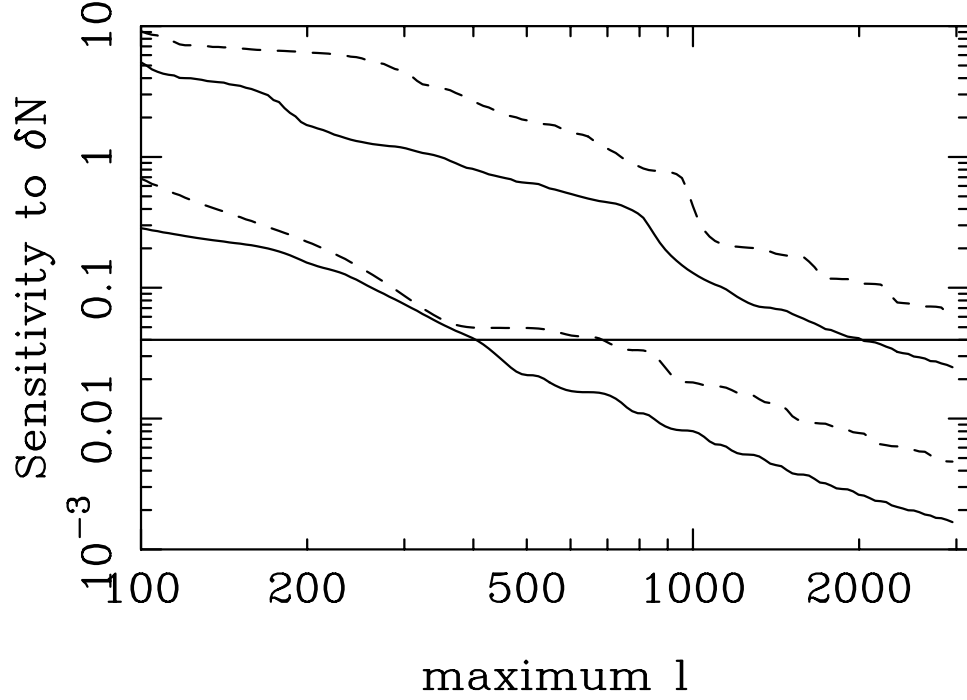Figure 2: Same as Fig. 1, but for the P_l's, the electric field polarization anisotropies.

Figure 3: One-σ sensitivity to δN_ν, for an experiment cosmic-variance limited up to some maximum multipole moment. The horizontal line, $\delta N_\nu = 0.04$, is the change in effective number of neutrino families due to neutrino heating and the QED effect. The bottom two curves are for the case where all cosmological parameters except N_ν are fixed, while the top curves represent the case where all parameters are determined simultaneously. For each group, the dashed line shows the results using only temperature anisotropy data, while the solid line shows the improvement obtained by including polarization data in the analysis.